

UNITED STATES NAVAL POSTGRADUATE SCHOOL



CONTINUOUS REVIEW INVENTORY POLICIES IN NAVY APPLICATION

by

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ABSTRACT:

Inventories exist to provide service to customers by satisfying their demands from on hand material. Variations on the traditional continuous review formulation are investigated in an effort to improve service as measured in terms of units short. It is proposed that "smaller" order quantities will improve service in current Navy supply operations. Basing holding costs on inventory position instead of on hand quantities does not provide significant or consistent reductions in order quantities. The desirability of smaller order quantities is not adequately tested by steady state analyses.

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1. Introduction

Inventories exist for a very simple reason: so that stock is available to meet customer demands when they occur. The alternative is to maintain no stock and to order the item from a supplier after the customer makes his demand. Carrying inventory requires an investment in facilities, organization, and in the stock itself. The inventory manager's basic problem is to maximize the level of service to customers while minimizing the investment required. As these two objectives are incompatible with each other, the best trade-off is continually being sought.

The costs associated with maintaining inventories are both easily understood and difficult to quantify. Few, if any, of the costs are actual accounting entries. In any case, the justification for incurring these costs is an ill-defined measure called "service," service to the customer. If no stocks were carried in inventory, a customer's demand could not be satisfied until the material was ordered and received. In some retail situations, even if no stock is carried, the lead times are sufficiently short to be negligible. The more common situation, however, is of unacceptably long lead times. Thus, inventories exist to provide timely service to customers.

Inventory control is exercised on a cyclic pattern of purchases of stock, the dwindling of stocks by demands placed on the inventory, and the subsequent reordering of stock. The inventory manager decides when and how much stock to purchase. When to order is usually expressed in terms of a reorder point, defined on some measure of the stock level. How much to buy is expressed in terms of the order quantity. Choice of the best values of the reorder point and order quantity depends upon the formulation and measure of effectiveness being used by the

decision maker. For continuous review inventory systems, specification of particular values for the reorder point and order quantity will be referred to as a policy for managing an item.

Prior to discussing inventory policies, we must discuss the nature of demand processes. The simplest situation is that of a constant demand for an item. An example is a situation where it is known that demand for the item will be, say, 25 units per week now and for a very long time into the future. In this situation the inventory manager can determine with perfect certainty the consequences of any inventory policy. However, there are few real-world inventories in which so much is known about the demand process.

Generally, demand can only be described as a random variable with a certain distribution and moments. For inventory control purposes one is usually interested in total demand during some period of time. If demand is a random variable, then the inventory manager cannot know in advance what the total demand in the next time period will be. Demand in the next time period can only be described in terms of its probability distribution, mean and variance. In terms of descriptive adjectives, words like "slow mover" or "active item" are commonly associated with low and high values of mean demand and words like "stable" and "erratic" are associated with low and high demand variances.

It follows that if demand is a random variable, then future states of the inventory system will not be known with certainty and may only be estimated or predicted in a probabilistic sense. It is theoretically impossible and/or economically infeasible to prevent all shortages when demand is a random variable. In a probabilistic inventory system a risk of shortage is always taken.

In this report we wish to examine continuous review inventory policies in the context of Navy supply operations. We have several objectives in doing so. The first objective is simply to present statements of the basic operating characteristics and interactions in continuous review policies. The second objective is to investigate a continuous review formulation that applies inventory holding costs to on-order assets as well as on-hand assets. This is done in the belief that such a formulation will produce "smaller" optimal order quantities, which in turn are believed desirable. The third objective is to bring up the question of whether the minimization of total variable costs formulations of classical inventory theory are the most appropriate for Navy inventory operations.

In the next section the basic arguments of classical inventory theory are given along with a discussion of continuous review policies and their relationship to the economic order quantity of deterministic inventory systems. The third section discusses characteristics of the environment of Navy inventories and the special problems encountered in their management. Section 4 develops the inventory position model and compares it to the normal formulation. The fifth section presents a comparison of classical and UICP inventory policies for a sample of eight items. In the final section we summarize and state some tentative conclusions.

2. Formulations

In what we shall refer to as classical inventory theory, the measure of effectiveness has always been the minimization of total variable inventory costs. These costs include ordering costs, holding

costs, and shortage costs. For continuous review policies the values of the decision variables, reorder point and order quantity, are then selected to achieve an economic balance between these costs in such a way as to minimize their sum. The techniques involved are straightforward. Variable costs are expressed as analytic functions of the decision variables. The total variable cost expression is then differenced or differentiated with respect to each decision variable and the resulting expressions are set equal to zero and solved for the optimum values of the decision variables.

We give expressions for the optimal reorder point and order quantity for deterministic and probabilistic continuous review policies. The following notation is used throughout the report:

- A = order cost in dollars;
- C = item unit cost in dollars;
- I = inventory holding rate as a percentage per year;
- δ = mean annual demand in units;
- μ = mean lead time demand in units;
- σ = standard deviation of lead time demand in units;
- τ = constant lead time in years;
- λ = shortage cost in dollars per unit short;
- $\Phi(r)$ = probability that lead time demand exceeds r ;
- r = reorder point; and
- Q = order quantity.

When demand is a constant, rather than a random variable, and shortages are avoided the optimal order quantity is

$$Q_{EOQ} = \left[\frac{2A\delta}{IC} \right]^{1/2}. \quad (2.1)$$

It is labeled Q_{EOQ} to denote it as the economic order quantity established by Wilson in 1915. Specification of the reorder point when demand and lead time are constant is trivial.

The next case is that of a probabilistic continuous review policy when lead time demand has a normal distribution and there is a fixed cost per unit short. Hadley and Whitin ⁽¹⁾ results are given here. The equations for the optimal order quantity and reorder point are:

$$Q_N = \left[\frac{2\delta[A + \lambda\alpha(r_N)]}{IC} + 2\beta(r_N) \right]^{1/2} \quad (2.2)$$

and

$$ICQ_N = \phi(r_N) [\lambda\delta - IC(r_N - \mu)] + ICQ_N \phi(r_N). \quad (2.3)$$

In the above equations

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}};$$

$$\Phi(r) = \int_r^{\infty} \phi(x) dx;$$

$$\alpha(r) = \sigma\phi\left(\frac{r-\mu}{\sigma}\right) - (r - \mu)\phi\left(\frac{r-\mu}{\sigma}\right); \text{ and}$$

$$\beta(r) = \frac{1}{2} \left[\sigma^2 + (r-\mu)^2 \right] \phi\left(\frac{r-\mu}{\sigma}\right) - \frac{\sigma}{2} (r-\mu) \phi\left(\frac{r-\mu}{\sigma}\right).$$

The variables in equations (2.2) and (2.3) are subscripted with N to indicate the normal treatment of holding costs; i.e., holding costs charged against on-hand quantities.

One can make a number of observations about equations (2.2) and (2.3); the first might be that they seem complex and difficult to use.

Neither equation yields one of the variables (Q_N or r_N) explicitly. The equations must be solved many times (iterated) until their joint solution is determined. At each step in the basic iteration, equation (2.3) must be solved numerically (iterating on r_N). Determining the optimal Q and r thus involves a bit of computation.

However, the appearance of equations (2.2) and (2.3) must not be allowed to obscure their basic message. Sacrificing some detail, the equations may be written as

$$Q_N = Q_{EOQ} + h \sqrt{\alpha(r_N)}, \quad h > 0; \quad (2.2a)$$

and

$$\phi(r_N) = \frac{ICQ_N}{ICQ_N + \lambda \delta} \quad (2.3a)$$

The non-negative quantity $\alpha(r_N)$ decreases to zero as r_N increases. Thus, equation (2.2a) shows that $Q_N \geq Q_{EOQ}$, and that $Q_N = Q_{EOQ}$ only if $r_N = +\infty$. Note that r_N is not restricted to take only non-negative values.

Equation (2.1) clearly shows that the order quantity decreases with increasing item cost. This result does not hold in equations (2.2) and (2.3). Consider the following example. We let $A = \$75$, $I = .18$ per year, $\delta = 1000$ units per year, $\lambda = \$3500$, $\mu = 1250$ units, and $\sigma = 400$ units. Q_{EOQ} and Q_N vary with unit costs, C , as shown in Figure 1. Note that Q_N actually increases with increasing unit cost, after about \$200 in the example given.

A further relationship of interest is that, in simplified form, the probability of being out of stock at any time instant may be written as

$$P_{out} = \frac{1}{Q_N} \alpha(r_N). \quad (2.4)$$

However, iteration of the type required to solve equations (2.2) and (2.3) is not feasible with the computer systems available at the inventory control points. Consequently, equations (2.2) and (2.3) are separated by determining the order quantity from some other criterion, leaving simply a variation of equation (2.3) to be solved once for the reorder point. To be perfectly correct about the order quantity, the document ALRAND 58⁽²⁾ states that UICP policies use the economic order quantity, Q_{EOQ} , subject to minimum and maximum order quantity constraints.

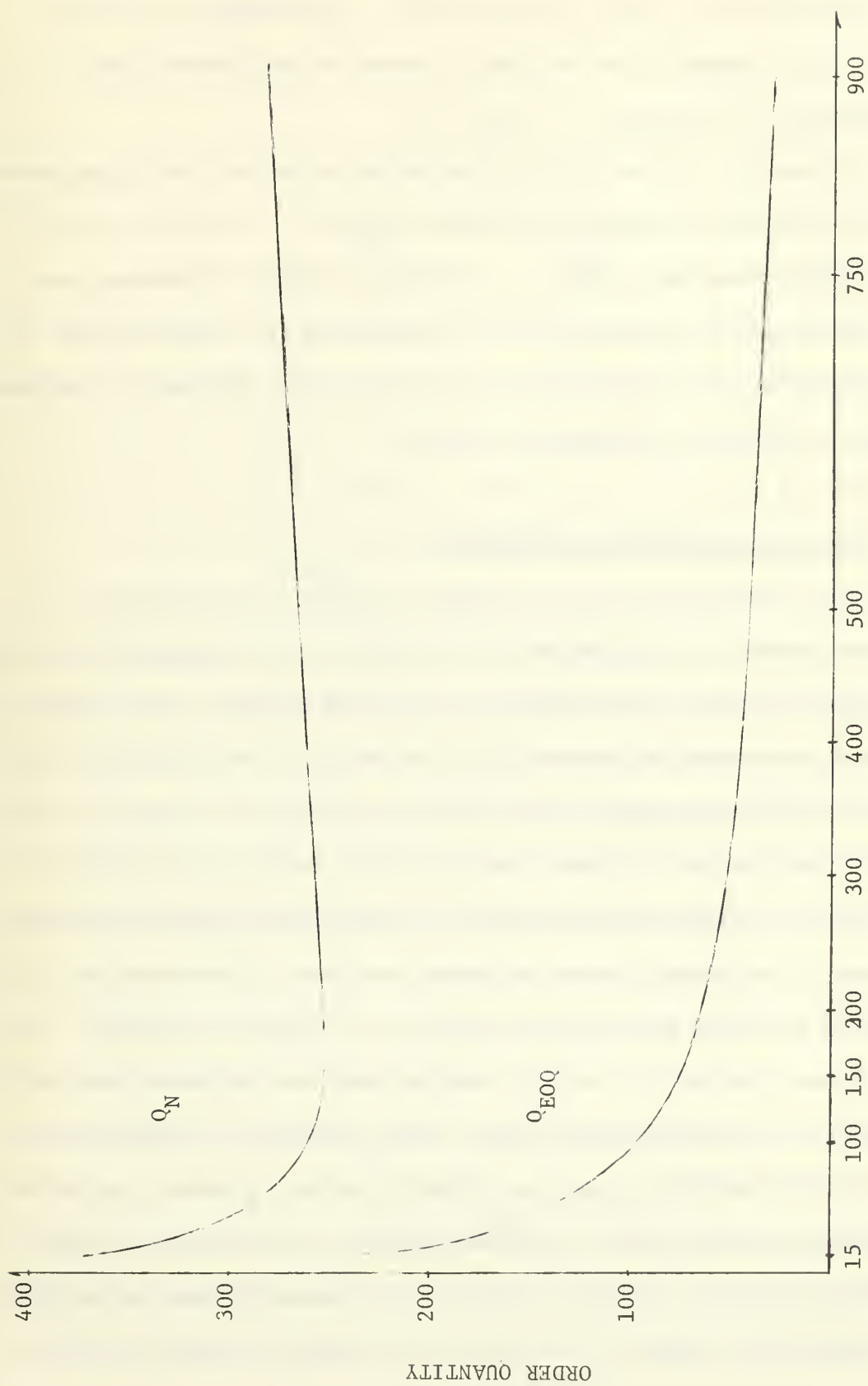
The determination of the reorder point, after the order quantity is determined, is made using equation (2.3a). The equation (2.3a) is derived from a minimization of total variable cost formulation, and the values of the cost parameters must be specified. A problem exists only in specifying the shortage cost parameter. Instead of treating customer service as a goal, the formulation treats the disservice created by a stockout as a cost. However, the shortage cost cannot be determined from the standpoint of customer disservice. Doing so would be a difficult problem in its own right. Even so, such a scheme would almost surely prove operationally infeasible in terms of the procurement funds requirements of the policies which would result. The severe procurement funds limitations typical of current operations force the decision makers to use the shortage cost parameter as a lagrange multiplier. The value of the shortage cost parameter is thus determined from funding feasibility considerations rather than from consideration of disservice to the customer. The feasible value of the lagrange multiplier may be interpreted as the value of the shortage cost as imputed by the procurement budget. Note that the imputed shortage cost so determined applies to all items within a given material cognizance class, regardless of cost or end use. Another point to note that in producing policies which

years. This, plus the fact that Congress feels that the Department of Defense must bend over backwards to help the small businessman, assures that no two ships are exactly alike in their equipment.

The second major characteristic which constrains the management of Navy inventories is the complex funding structure. Navy managed material is controlled by three major inventory control points and a number of project offices. Individual items are grouped into material cognizance classes (cogs) and each cog has a single manager, though a given manager will generally manage many cogs. Funds with which to procure stock items are provided to each individual cog. The procurement funds allocated to a cog cannot be used for any other purpose. The amount of these funds is not controlled by the inventory manager and history has shown that the inventory manager has not been overly successful in demonstrating the relationship between funding and supply effectiveness to the budget people.

The inventory manager runs his organization with operations and maintenance funds which are separate and distinct from cog procurement funds, and which may not be spent for any other purpose. The level of O&M funding determines, in some way, the size of the organization's labor force and hence puts an upper limit on the number of procurement orders which can be processed by the organization in any period of time. O&M funding is tight enough that reorder workload becomes a real constraint on inventory policies.

The tremendous numbers of items to be managed, together with relatively limited computation facilities, dictates that Navy inventory policies be of simplified form. The Uniform Inventory Control Point (UICP) policies are based on probabilistic continuous review policies.



ITEM UNIT COST IN DOLLARS

FIGURE 1. Q_{EOQ} AND Q_N AS FUNCTIONS OF UNIT COST

If it is desired to decrease P_{out} , this may be accomplished by either (1) increasing the order quantity, or (2) increasing the reorder point. The changes to be made are dictated by the interactions represented in equations (2.2) and (2.3).

In summary, the two decision variables in probabilistic continuous review policies are inescapably linked together. A change in one of the variables causes a change in the other variable. Shortages can be reduced (and service improved) by increasing the reorder point, or by increasing the order quantity. In either case, decreasing shortages requires increased investments in stock.

3. Inventory Management in the Navy

Navy inventories exist to provide service to fleet customers and possess a number of characteristics which make their management difficult at best. The first characteristic is that the range of items carried in Navy inventories is greater than that of any other enterprise. The Navy supply system manages almost 800 thousand different items. At least three quarters of these items are "slow movers" with erratic, difficult to predict demand patterns. Most private enterprises would not stock this range of items and would not stock slow movers at all. The Navy is forced to carry slow movers for a number of reasons. In many cases, the Navy is the only user of the item and spares are not available on an off-the-shelf basis. The alternative to not stocking the item is possibly having a ship tied up until a contract can be let and the item manufactured. Another reason for the extensive range of items which must be stocked is the lack of commonality and standardization among fleet units. Fleet units vary in age from one to over 25

Equations (4.1) and (4.2) are derived from the total cost equation

$$K_M = \frac{A\delta}{Q_M} + IC \left(r_M + \frac{Q_M + 1}{2} \right) + \frac{\lambda\delta}{Q_M} \alpha(r_M). \quad (4.3)$$

The reorder point equation (4.2) and the total cost equation (4.3) may present several difficulties. The first of these difficulties is that equation (4.2) need not always make sense. $\phi(r_M)$ is a probability but

there is no guarantee that $\frac{ICQ_M}{\lambda\delta} \leq 1$. If $\frac{ICQ_M}{\lambda\delta} > 1$, it means simply that expressions (4.1) and (4.2) do not have a common solution. In this case the order quantity and reorder point may be determined from a search of the cost surface generated in the three dimensions Q , r , and K . (At one time UICP policies were based on inventory position and the reorder point was determined from the equation

$$\phi(r) = \frac{ICQ_{EOQ}}{\lambda\delta}.$$

However, when used for low demand, high cost items the equation did not always make sense and they had no alternative for this contingency.)

The cost surface is generated by evaluating equation (4.3) for a range of values of Q and r . In doing so, though, one must be careful of the holding cost term. Trouble with the reorder point equation (4.2) occurs when shortages are frequent and the reorder point is very low. It is possible for the reorder point to be so low (negative) that the term $(r + \frac{Q+1}{2})$ is negative. Care must be exercised to assure that the holding cost term does not go negative.

To summarize, if holding costs are applied to inventory position, the optimal order quantity and reorder point are determined from equations (4.1) and (4.2). If the iteration process produces trial Q_M values which violate equation (4.2) when substituted into that equation, Q_M and r_M must be determined directly from the total cost equation (4.3).

can also be applied to on-order assets. The risk of obsolescence (a sharp drop in the demand rate to zero) at some time in the future is most serious for the most recently purchased stocks, especially those on order.

Having given the motivation for applying holding costs to the inventory position, it remains to structure the model, indicate the optimal order quantity and reorder point equations, and to make comparisons. The total cost equation is composed of three terms: ordering costs, holding costs, and shortage costs. Applying holding costs to inventory position instead of to on-hand inventory will cause a change only in the holding cost term. As defined earlier,

$$IP = OH + OO - BO.$$

From reference (1) it may be seen that

$$E(OH) = r + \frac{Q+1}{2} + B(Q,r) - \mu$$

and

$$E(IP) = r + \frac{Q+1}{2},$$

where $B(Q,r)$ represents the expected number of backorders at any moment in time. One point to note is that $E(OH)$ and $E(IP)$ differ by only a constant, μ , only if the $B(Q,r)$ term is zero, or by very little other than a constant if shortages are rare. Hence different policies will result from using inventory position for items whose $B(Q,r)$ terms are large.

The equations for the optimal order quantity and reorder point for the modified system are:

$$Q_M = \left[\frac{2\delta[A + \lambda\alpha(r_M)]}{IC} \right]^{1/2} \quad (4.1)$$

and

$$\Phi(r_M) = \frac{ICQ_M}{\lambda\delta} \quad (4.2)$$

are feasible within the procurement budget, only the reorder points are manipulated. Thus if the budget for the next period is quite restrictive the reorder points would be lowered and assets eaten down.

As mentioned above, the level of procurement funding has been quite low since prior to the build-up in Southeast Asia. Correspondingly, material availability has been marginal (or disgraceful, depending upon your viewpoint) and criticism of UICP policies has been high. In FY68 point-of-entry availability in 1H cog was 37%⁽³⁾. Even in "well-to-do" cogs like SPO's 6H, point-of-entry availability has not been high; 72% in 6H cog in FY68. UICP procedures have had to dictate negative safety levels and even perpetual backorder situations for segments of the inventory.

In an attempt to modify UICP policies so as to make them more responsive to the conditions created by limited procurement budgets, a model was structured in which holding costs were applied to the item inventory position instead of the item on-hand inventory level. This model is described in the next section of the report.

4. The Inventory Position Model

The simple-minded motivation for constructing this model was that, intuitively, if funds are scarce, the inventory manager ought to spend procurement funds in lots of little purchases rather than in a few big ones. Recall that the situation we are describing involves the allocation of a single budget to the procurement of many thousands of items. Item demand is uncertain and the funds must be spent on the items which will move. Since lead times are long and actual demand cannot be known in advance, one would like to hedge by using small order quantities. Such policies would minimize the short term "bad buy" problem. A

number of mechanisms exist to generate smaller order quantities than are produced by equations (2.2) and (2.3). One mechanism was thought to be that of increasing holding costs by charging them against inventory position instead of on-hand inventory. The desirability of smaller order quantities and the use of inventory position to achieve this end was suggested by Lt. John M. Cook, SC, USN ⁽⁴⁾.

Inventory position, IP, is defined as the quantity on hand, OH, plus the quantity on order, OO, minus the quantity backordered, BO. It follows that $IP \geq OH$, and that holding costs will be greater if IP is used than if OH is used. The greater holding costs should make the optimal order quantity smaller and achieve the desired results; so says the intuition, but such was not the case, as will be explained.

A case can be made for computing holding costs from the inventory position. The Navy inventory holding rate has three components: storage, interest, and obsolescence. The magnitudes of these factors are approximately 1, 4, and 10, respectively. Storage costs (facilities, heat and light, and security) are most naturally associated with stocks physically on hand, but it can also be argued that these costs are fixed or relatively insensitive to actual on-hand stock levels. The interest factor is used to reflect the loss of opportunity of other uses of the capital invested in inventory stock; i.e., a reflection of the scarcity of funds and the need to allocate them wisely. On-hand stock represents dollars spent, material bought and paid for. The proper time to ration procurement funds is much earlier, when orders are placed. Although contracts can be canceled, the Navy's use of the terms committed and obligated are quite descriptive. It seems clear that the interest factor could reasonably apply to on-order assets. The third factor, obsolescence,

Direct comparison of the normal and modified order quantities and reorder points is difficult, so a number of example solutions were generated for comparison purposes. From a sample of eight items, it seems that the normal and modified calculations produce nearly identical results. The calculations were performed using eight 1H cog FSN's. An order cost of \$42 was assumed as was a holding rate of 15% per year. Item cost and demand data were obtained from SPCC item data files. A shortage cost of \$10 was used and is the value used in UICP calculations of September, 1967, for 1H cog. Results of the normal and modified calculations are shown in Table 1, along with information about the items. The sample includes unit costs from \$1.66 to \$1040, annual demand rates from 80 to 95,000 units, lead times from 2.4 to 4.0 quarters, and coefficients of lead time variation of 0.19 to 0.36. The computations required to produce Table 1 were programmed for an IBM 360/67 system by Lt. W. Dean Free, SC, USN.

Inspection of Table 1 shows that the modified equations succeeded in reducing the order quantities in only three of the eight cases and that $Q_M > Q_N$ in the remaining five cases. The conclusion is made that policies based upon inventory position do not differ consistently and/or measurably from policies based upon on-hand inventory levels. The notion of reducing order quantity size to hedge against future uncertainties cannot be implemented by charging holding costs against inventory position.

Still the idea of reducing order quantity size in a climate of low procurement funding seems sound. We note that this idea occurred to Prichard. (5) His memorandum describes the influence of higher inventory holding rates on order quantities, ordering rates, and

Table 1. Normal and Modified Order Quantities and Reorder Points

FSN	§C	μ	σ	τ	O _M	O _N	r _M	r _N
4820-965-1139	48.30	3,326	652	1.00	738	713	3,972	4,018
4710-277-4656	1.66	59,293	16,154	0.63	12,305	12,401	103,292	103,251
4710-289-3513	3.06	45,349	12,984	0.85	9,557	9,630	76,501	76,479
5925-258-2945	53.30	713	257	0.60	333	315	907	937
4940-261-8415	26.00	1,676	424	0.73	487	480	2,265	2,279
6140-731-9144	3.66	16,710	3,923	0.73	3,575	3,590	26,073	26,073
2010-344-0728	456.00	127	36	0.60	116	290	-57	-132
4220-300-9929	1040.00	60	14	0.75	181	107	-92	-41

effectiveness measures. In the next section we analyze the effect of restricted order quantities on various operating measures.

5. Comparison of Normal and Restricted Order Quantities.

As noted in Section 3, the Navy UICP inventory control procedures decouple the order quantity and reorder point computations. The order quantity is first determined and then the reorder point is computed from

$$\phi(r) = \frac{ICQ}{ICQ + \delta\lambda} \quad (2.3a)$$

ALRAND 58⁽²⁾ states that the order quantity shall be the economic order quantity, $Q = [2A\delta/IC]^{1/2}$. However it is believed that certain overrides on the order quantity are employed such that the order quantity will never be less than the greater of one unit or one month's expected demand. The order quantity so determined is generally smaller than would be dictated by the normal solution as represented in equations (2.2) and (2.3).

The comparison of normal and restricted order quantity policies in this section is based upon the eight 1H cog items used in Section 4. Three sets of inventory policies will be evaluated:

Set 1. Normal continuous review inventory policies based upon equations (2.2) and (2.3);

Set 2. UICP continuous review policies with

$$Q = \max\left\{\left[\frac{2A\delta}{IC}\right]^{1/2}, 1, \delta/12\right\}$$

and the reorder point equation (2.3a); and

Set 3. UICP continuous review policies with

$$Q = \max\left\{\left[\frac{2A\delta}{IC}\right]^{1/2}, 1, \delta/4\right\}$$

and the reorder point equation (2.3a).

Sets 2 and 3 differ in the order quantity computation; Set 2 uses a minimum order quantity equal to one month's demand while Set 3 uses a minimum order quantity of three month's demand.

Each set of policies is evaluated in terms of:

- order quantities and reorder points
- item risk, in the UICP sense, defined as the probability that lead time demand exceeds the quantity on hand to meet demand
- P_{out} , the probability of being out of stock (on hand quantity equal zero) at any moment in time
- the total number of units short per year
- the total average safety stock investment
- the total average operating stock investment
- the total average investment
- the total reorder workload
- the total variable inventory costs, excluding the cost of the items purchased.

Average safety stock investment for the i^{th} item is defined as $\max\{0, c_i(r_i - \mu_i)\}$. Average operating stock investment for the i^{th} item is defined as $c_i(\frac{Q_i}{2})$ if $r_i - \mu_i + Q_i > 0$, and zero otherwise. The reorder workload for the i^{th} item is defined as δ_i/Q_i . Total variable inventory cost exclusive of procurement costs are computed in the normal sense as the sum of ordering costs, holding costs, and shortage costs. For the order quantities and reorder points of each set of policies the variable inventory cost K is computed from

$$K = \frac{\lambda \delta}{Q} + IC \left[r - \mu + \frac{n+1}{2} + \beta(r) \right] + \frac{\lambda \delta}{Q} \alpha(r) , \quad (5.1)$$

where $\alpha(r)$ and $\beta(r)$ are defined as in Section 2. Equation (5.1) produces the optimal order quantity and reorder point equations (2.2) and (2.3).

The probabilistic measures are defined as follows. In UICP documentation, risk is defined as

$$\int_r^{\infty} f(x)dx ,$$

where $f(x)$ is the density function of lead time demand. The probability that the on hand quantity of a given item is zero at any moment in time, P_{out} , is defined as

$$P_{out} = \frac{1}{Q} \int_r^{\infty} (x-r)f(x)dx .$$

If lead time demand is approximated with a normal distribution, then $P_{out} = \frac{\alpha(r)}{Q}$. Finally the total number of units short per year is simply δP_{out} . The P_{out} measure is more useful than the simple risk measure since the risk measure ignores the procurement cycle length. It matters a great deal whether a given risk is incurred once every two weeks or once every two years.

Comparisons of the three sets of policies is shown in Table 2. Inventory theory tells us that the policies of Set 1 are the best; they are the only policies derived directly from a cost minimization formulation. The policies of Set 2 produce the lowest total units short per year measure, only 85% of the number produced by the policies of Set 1. However to achieve this reduction in units short, Set 2 policies require a 7.3% greater total investment and a reorder workload which is almost three times higher. Set 3 policies produce virtually the same units short figure with the same reorder workload as Set 1 and only a 2.7% greater investment than Set 1. In terms of total variable inventory costs exclusive of procurement costs Set 1 is the best (as it must be), but Set 3 is only 2.7% higher. As argued much earlier, the variable cost components are not accounting journal entries anyhow.

ITEM DATA

SET 1

<u>FSN</u>	<u>COST</u>	<u>ANNUAL DEMAND</u>	<u>LEAD TIME (YRS)</u>	<u>Q</u>	<u>r</u>	<u>RISK</u>	<u>P_{OUT}</u>	<u>UNITS SHORT</u>
-1139	48.30	3,326	1.00	713	4,018	.140	.068	226
-4656	1.66	94,869	0.63	12,401	103,251	.003	.001	95
-3513	3.06	53,058	0.85	9,630	76,479	.008	.004	186
-2945	53.30	1,184	0.60	315	937	.190	.087	103
-8415	26.00	2,296	0.73	480	2,279	.080	.031	71
-9144	3.66	22,893	0.73	3,590	26,073	.008	.003	69
-0728	456.00	211	0.60	290	-132	1.000	.893	188
-0728	1040.00	80	0.75	107	-41	1.000	.944	<u>76</u>
TOTAL UNITS SHORT PER YEAR								1014
SAFETY STOCK INVESTMENT				\$263,537				
OPERATING STOCK INVESTMENT				<u>63,500</u>				
TOTAL INVESTMENT								\$327,037
WORKLOAD IN ORDERS PER YEAR								34
VARIABLE INVENTORY COSTS								\$61,701

SET 2					SET 3				
<u>Q</u>	<u>r</u>	<u>RISK</u>	<u>P_{OUT}</u>	<u>UNITS SHORT</u>	<u>Q</u>	<u>r</u>	<u>RISK</u>	<u>P_{OUT}</u>	<u>UNITS SHORT</u>
277	4,356	.057	.058	191	831	3,994	.153	.062	206
7,905	105,978	.002	.001	114	23,717	100,001	.006	.001	114
4,421	80,146	.004	.003	165	13,264	74,952	.011	.004	223
112	1,093	.070	.069	81	296	962	.166	.077	91
223	2,439	.036	.027	62	537	2,250	.088	.032	74
1,907	26,988	.004	.003	64	5,721	25,419	.013	.003	66
17	140	.355	.517	109	52	115	.627	.184	39
7	57	.577	1.000	<u>80</u>	20	48	.795	.676	<u>54</u>
				866					867
\$317,362					\$250,496				
<u>33,534</u>					<u>85,426</u>				
				\$350,896					\$335,922
				93					32
				\$66,943					\$63,369

The obvious question, of course, is which set of policies a Navy inventory manager would choose for these eight items. We suspect that the policies of Set 3 would be chosen over the theroretically best policies of Set 1. This suggests that the minimization of variable inventory costs may not be the best criterion for continuous review inventory policies for the Navy. Rejection of this criterion is not appropriate either, though. The order quantities of the policies of Set 3 were set equal to quarterly demand, but the reorder points were determined from equation (2.3a) which is derived from a cost minimization formulation. The cost minimization formulation cannot be discarded until a substitute formulation is proposed and evaluated.

An alternative formulation of continuous review policies for Navy multi-item inventories was recently proposed by Lt. A. P. Tully, SC, USN⁽⁶⁾. His formulation is based on the following simple analysis of conditions. The inventory manager's goal is to maximize effectiveness by minimizing shortages. But the Navy inventory manager has two formidable constraints: a limit on procurement funds and a limit on the number of procurement orders which can be processed in any period of time. The formulation is then simply

to: minimize total units short

subject to: (1) total procurement costs $\leq$ procurement budget

(2) total number of reorders $\leq$ reorder workload constraint.

Note that the formulation is void of the traditional ordering, holding, and shortage cost parameters.

A completely general solution for the above formulation was not obtained but a computationally feasible solution was obtained for the simplified problem created by determining order quantities from the reorder workload constraint. In the simplified formulation the reorder

workload constraint is assumed to be active and this in turn uniquely determines the order quantities. This simplification leaves shortages a function of only one set of variables, the reorder points, and shortages are to be minimized subject to only one constraint, procurement dollars.

Miller⁽⁷⁾ has recently formulated a provisioning model along the lines taken by Tully. Miller's problem is that of determining initial provisioning quantities for a class of items subject to a single provisioning budget for the class of items. The "minimize shortages subject to constraints" formulation seems to be an excellent alternative to cost minimization formulations for inventory control, so long as the resulting mathematical programming problems have computationally efficient solutions.

6. Conclusion

The original purpose of this report was to investigate inventory policies in which holding costs were based on inventory position. It was believed (1) that use of inventory position would produce "smaller" order quantities, and (2) that smaller order quantities are desirable in the climate created by limited procurement authority. Section 4 indicated that inventory position order quantities do not differ significantly and/or consistently from normal order quantities.

The desirability of smaller than normal order quantities was partially investigated in Section 5. A priori goodness in order quantity size should be judged from its influence on shortages. The policies of Sets 2 and 3 represent the extremes in order quantity size in most cases. The total units short per year measure are nearly identical while the smaller order quantities of Set 2 require an

increased investment and greatly increased reorder workloads. The influence of the limitation on procurement funds is represented in each case through the shortage cost parameter λ . On the basis of the analysis provided in Section 5 it appears that smaller order quantities are not advantageous.

However before such a conclusion is reached about smaller order quantities it is noted that the analysis in Section 5 was a steady state analysis descriptive of long run average operations. The Navy budget cycle is one year and short term effects are important. In other words, the steady state analysis in Section 5 is not adequate to judge the desirability of smaller order quantities. In the short term demand on many items exhibits large variations. Smaller order quantities should allow the manager to better match funds spent with material demanded.

We have also suggested that it is time to investigate new formulations and new criteria for continuous review inventory policies. The traditional cost minimization formulation yields operationally unacceptable policies in high unit cost-low shortage cost situations typical of parts of the Navy inventory.

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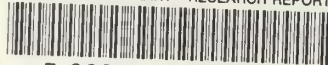
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